GCE AS MARKING SCHEME

SUMMER 2018

AS (NEW)
FURTHER MATHEMATICS
UNIT 1 FURTHER PURE MATHEMATICS A
2305U10-1

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Further Mathematics – AS Unit 1 Further Pure Mathematics A

SUMMER 2018 MARK SCHEME

Qu	Solution	Mark	Notes
1. a)	det B = 0	B1	
b)	i) det A = -10 $A^{-1} = \frac{-1}{10}\begin{pmatrix}-3 & -2\\ 1 & 4\end{pmatrix}$	B1 M1 A1	
	ii) $X = \frac{-1}{10}\begin{pmatrix}-3 & -2\\ 1 & 4\end{pmatrix}\begin{pmatrix}-4\\ 1\end{pmatrix}$ $X = \begin{pmatrix}-1\\ 0\end{pmatrix}$	M1 A1	 cao
2.	When n = 1, LHS = 1x4 = 4 and RHS = 1/3 x 1 x 2 x 6 = 4 Therefore, expression is valid for n = 1. Assume result is true for n = k i.e. $\sum_{r=1}^{k} r(r+3) = \frac{1}{3}k(k+1)(k+5)$ Consider n = k+1 $\sum_{r=1}^{k+1} r(r+3) = \frac{1}{3}k(k+1)(k+5) + (k+1)(k+4)$ $= \frac{1}{3}(k+1)[k(k+5) + 3(k+4)]$ $= \frac{1}{3}(k+1)[k^2 + 8k + 12]$ $= \frac{1}{3}(k+1)(k+2)(k+6)$ $= \frac{1}{3}(k+1)\big((k+1)+1\big)((k+1)+5)$ If expression is true for n = k, it's also true for n = k+1. As it's true for n = 1, by mathematical induction, it's true for all positive integers n.	B1 M1 M1 A1 A1 E1	 Award for a perfect solution including the last line.

Qu	Solution	Mark	Notes
3. a)	METHOD 1 Because $\alpha\beta\gamma = 0$, put $\alpha = 0$ Then, $\beta + \gamma = -9$ and $\beta\gamma = 20$ Attempt to solve, $\beta = -4$ and $\gamma = -5$	B1 B1 M1 A1	Accept solutions where variables are interchanged throughout
	METHOD 2 Because $\alpha\beta\gamma = 0$, either $\alpha = 0$ or $\beta = 0$ or $\gamma = 0$ $ax^3 + bx^2 + cx + d = 0$ $x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$ $x^3 + 9x^2 + 20x = 0$ $x(x^2 + 9x + 20) = 0$ $x(x+4)(x+5) = 0$ $\therefore x = 0, x = -4, x = -5$ $\alpha = 0$, $\beta = -4$ and $\gamma = -5$	(B1) (B1) (M1) (A1)	
b)	METHOD 1 If $\alpha = 0$, $\beta = -4$ and $\gamma = -5$ $3\alpha = 0$, $3\beta = -12$ and $3\gamma = -15$ Therefore $x(x+12)(x+15) = 0$ Expanding, $(x^2 + 12x)(x + 15) = 0$ or $(x^2 + 15x)(x + 12) = 0$ or $x(x^2 + 27x + 180) = 0$ $\therefore x^3 + 27x^2 + 180x = 0$	B1 M1 A1 A1	
	METHOD 2 $3\alpha.3\beta.3\gamma = 27\alpha\beta\gamma = 0\left(= \frac{-d}{a}\right)$	(B1)	
	$3\alpha + 3\beta + 3\gamma = 3(\alpha + \beta + \gamma) = -27\left(= \frac{-b}{a} \therefore \frac{b}{a} = 27\right)$ OR	(M1)	M1 for attempting either method
	$9\alpha\beta + 9\beta\gamma + 9\gamma\alpha = 9(\alpha\beta + \beta\gamma + \gamma\alpha) = 180\left(= \frac{c}{a}\right)$	(A1)	A1 for both correct
	$\therefore x^3 + 27x^2 + 180x = 0$	(A1)	For use of their values above
4 (a)	(i) $\lvert z\rvert = \sqrt{(-3)^2 + 4^2} = 5$ $\tan^{-1}\frac{4}{-3} = -0.927$ or $\tan^{-1}\frac{4}{3} = 0.927$ $\arg(z) = 2.21$ $\therefore z = 5(\cos 2.21 + i\sin 2.21)$	B1 B1 B1	Accept awrt 2.2
	(ii) $\bar{z} = 5(\cos(-2.21) + i\sin(-2.21))$	B1	Accept 4.07 FT (i)
b)	$\lvert zw\rvert = 5 \times \sqrt{5}$ $(= 5\sqrt{5})$ $\arg(zw) = 2.21 + 2.68 = 4.89 \text{ or} -1.39$ $\therefore zw = 5\sqrt{5}(\cos(-1.39) + i\sin(-1.39))$	M1 M1 A1	FT (a) Accept awrt – 1.4 or 4.9

Qu	Solution	Mark	Notes
5. a)	$\frac{2}{n-1} - \frac{2}{n+1} = \frac{2(n+1)-2(n-1)}{n^2-1} = \frac{4}{n^2-1}$	B1	Convincing
b)	$\sum_{r=2}^{n} \frac{4}{(r^2-1)}$ $= \left(2 - \frac{2}{3}\right) + \left(1 - \frac{2}{4}\right) + \left(\frac{2}{3} - \frac{2}{5}\right) + \left(\frac{2}{4} - \frac{2}{6}\right) + \cdots +$ $+ \left(\frac{2}{n-3} - \frac{2}{n-1}\right) + \left(\frac{2}{n-2} - \frac{2}{n}\right) + \left(\frac{2}{n-1} - \frac{2}{n+1}\right)$ $= 2 + 1 - \frac{2}{n} - \frac{2}{n+1}$ $= 3 - \frac{2(n+1)+2n}{n(n+1)}$ $= \frac{3n(n+1)-2(n+1)-2n}{n(n+1)}$ $= \frac{3n^2-n-2}{n(n+1)}$ $= \frac{(3n+2)(n-1)}{n(n+1)}$ c.a.o	M1 A1 A1 m1 A1 A1	3 correct brackets Correct 1st, last and one other bracket si For common algebraic denominator Correct simplification of their numerator (at least quadratic)
c)	The 1st term is undefined	B1	oe
6. a)	$(1-2i)^2 = 1 - 2i - 2i - 4 = -3 - 4i$ $(1-2i)^3 = 1 - 6i - 12 + 8i = -11 + 2i$ $\therefore (-11+2i) + 5(-3-4i) - 9(1-2i) + 35$ $= 0$ therefore $1 - 2i$ is a root.	B1 M1 A1	B0 no working Substitution into cubic
b)	If $1-2i$ is a root then $1+2i$ is also a root. Method 1 $(x-1+2i)(x-1-2i) = x^2 - 2x + 5$ $x^3 + 5x^2 - 9x + 35 = (x^2 - 2x + 5)(x+7) = 0$ Final root is $x = -7$ Method 2 Product of roots = -35 OR sum = -5 $(1-2i)(1+2i)\alpha = 5\alpha = -35$ OR 1 – 2i + 1 + 2i + α = -5 Final root is $x = -7$	B1 M1 A1 A1 (M1) (A1) (A1)	 Unsupported answer M0
7. a)	Putting $z = x + iy$ $\lvert x + iy - 4 - i\rvert = \lvert x + iy + 2\rvert$ $\lvert(x-4) + i(y-1)\rvert = \lvert(x+2) + iy\rvert$ $(x-4)^2 + (y-1)^2 = (x+2)^2 + y^2$ $x^2 - 8x + 16 + y^2 - 2y + 1 = x^2 + 4x + 4 + y^2$ $12x + 2y - 13 = 0$	 M1 A1 m1 A1	 oe
b)	It is the perpendicular bisector of the line joining the points (4, 1) and (-2, 0) OR The locus of P is all the points which are equidistant from (4,1) and (-2,0).	B1 (B1)	

Qu	Solution	Mark	Notes
8. a)	Translation matrix: $\begin{pmatrix}1 & 0 & -1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}$	B1	
	Reflection matrix: $\begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix}$	B1	
	$T=\begin{pmatrix}0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & -1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}$	M1	
	$T=\begin{pmatrix}0 & 1 & 1\\1 & 0 & -1\\0 & 0 & 1\end{pmatrix}$	A1	Multiplying the wrong way gives $T=\begin{pmatrix}0 & 1 & -1\\1 & 0 & 1\\0 & 0 & 1\end{pmatrix}$
b)	$T=\begin{pmatrix}0 & 1 & 1\\1 & 0 & -1\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}=\begin{pmatrix}x\\y\\1\end{pmatrix}$	M1	FT their T from (a)
	Giving, y +1 = x and x – 1 = y Therefore the line of fixed points is $y=x-1$.	A1	Must be identical equations if FT
c)	$T^2=\begin{pmatrix}0 & 1 & 1\\1 & 0 & -1\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}0 & 1 & 1\\1 & 0 & -1\\0 & 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$	M1 A1	FT (a) At least 4 correct entries.
	$T^{-1}=\begin{pmatrix}0 & 1 & 1\\1 & 0 & -1\\0 & 0 & 1\end{pmatrix}$	B1	Accept $T^{-1}=T$

Qu	Solution	Mark	Notes
9.a)	(i) **AB** = (–2**i** + **j**) – (**i** + 2**j** – 3**k**) = –3**i** – **j** +3**k** Therefore, **r** = **i** + 2**j** – 3**k** + λ(–3**i** – **j** +3**k**) **r** = (1 - 3λ)**i** + (2 – λ)**j** + (-3 + 3λ)**k** (ii) $\frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z+3}{3}$	B1 M1 A1 B1	 Accept equivalent convincing
b)	If intersecting, 1 - 3λ = 2, 2 – λ = –4 + 4μ, –3 + 3λ = 7μ Solving (first pair), λ = -1/3 and μ = 19/12 Verifying in third equation: –4 ≠ 133/12 → they do not intersect	 M1 m1 A1 A1 E1	 Accept solution which use different pair: 2nd and 3rd equation give λ=54/19, μ=15/19 then -143/19≠2 1st and 3rd equation give λ= - 1/3, μ= - 4/7 then 7/3 ≠ - 44/7
c)	Let **n** = p**i** + q**j** + r**k** Then, from L_1 (p**i** + q**j** + r**k**).(–3**i** – **j** +3**k**) = 0 → –3p – q + 3r = 0 And, from L_2 (p**i** + q**j** + r**k**).(4**j** + 7**k**) = 0 → 4q + 7r = 0 Let r = t, then q = –7t/4 and p = 19t/12 **n** = (19t/12)**i** – (7t/4)**j** +t**k** (with t ≠ 0)	 M1 A1 A1 M1 A1	 Accept M1 here if not awarded previously Accept eg. If t = 12, **n** = 19**i** – 21**j** + 12**k**